

Extension 1 Miscellaneous 3 Worksheet



1. Show that the principal value of $\cot^{-1}(-\sqrt{3}) = \frac{5\pi}{6}$
2. By using the substitution $t = \tan \theta/2$, or otherwise, show that $(1 - \cos \theta)/\sin \theta = \tan \theta/2$
3. Let $f(x) = 2\cos^{-1}x$, Sketch the graph of $y = f(x)$, indicating clearly the coordinates of the endpoints of the graph. Also state the range of $f(x)$.
4. The polynomial $P(x) = x^2 + ax + b$ has a zero at $x = 2$. When $P(x)$ is divided by $x + 1$, the remainder is -15 . Find the values of a and b .
5. A skydiver jumps from a hot air balloon which is 2000 metres above the ground. The velocity, v metres per second, at which she is falling t seconds after jumping is given by $v = 50(1 - e^{-0.2t})$
 - (i) Find her acceleration ten seconds after she jumps. Give your answer correct to one decimal place.
 - (ii) Find the distance that she has fallen in the first ten seconds. Give your answer correct to the nearest metre.
6. A particle is moving so that acceleration $f''x = 18x^3 + 27x^2 + 9x$.
If, initially $x = -2$ and the velocity, v , is -6 . show that $v^2 = 9x^2(1 + x)$
7. A particle is undergoing simple harmonic motion on the x -axis about the origin. It is initially at its extreme positive position. The amplitude of the motion is 18 and the particle returns to its initial position every 5 seconds.
 - (i) Write down an equation for the position of the particle at time t seconds.
 - (ii) How long does the particle take to move from a rest position to the point halfway between that rest position and the equilibrium position?
8. Show that the values of x for which $f(x) = [x(x - 2)]^2$ is an increasing function are at the interval $0 < x < 1$, and $x > 2$. Also, find the points on the curve where the tangent is parallel to the x -axis.
9. Show that the right circular cylinder, open at the top, and of given surface area and maximum volume is such that its height is equal to the radius of the base.