Extension 1 Miscellaneous 2 Worksheet

1. Use mathematical induction to prove that $7^{2n-1} + 5$ is divisible by 12, for all integers $n \geq 1$.

2. Use mathematical induction to prove that, for integers $n \geq 1$,
   $1\times3+2\times4+3\times5+\ldots\ldots+ n(n+2) = n(n+1)(2n+7)/6$

3. Find the principal value of $\cot^{-1}(\sqrt{3})$.

4. Prove that $\tan^{-1} 1/2 + \tan^{-1} 1/3 = \pi/ 4$.

5. Let $f(x) = \sin^{-1}(x)$.

(i) State the domain and range of the function $f(x)$.

(ii) Find the gradient of the graph of $y = f(x)$ at the point where $x = -5$.

(iii) Sketch the graph of $y = f(x)$.

6. Use the fact that $\tan(\alpha - \beta) = (\tan\alpha - \tan\beta)/(1 + \tan\alpha\tan\beta)$ and

   (i)Show that $1+ \tan n\theta\tan(n+1)\theta = \cot\theta(\tan(n+1)\theta - \tan n\theta)$
   (ii) Use mathematical induction to prove that, for all integers $n \geq 1$,
   $\tan\theta\tan 2\theta + \tan 2\theta\tan 3\theta + \ldots + \tan n\theta\tan(n+1)\theta = -(n+1)) + \cot\theta\tan(n+1)\theta$

7. The graphs of the line $x + 3y - 7 = 0$ and the curve $y = x^3 + 1$ intersect at (1, 2). Find the exact value, in radians, of the acute angle between the line and the tangent to the curve at the point of intersection.

8. One solution of the equation $2\cos 2x = x + 1$ is close to $x = 0.4$. Use one application of Newton's method to find another approximation to this solution. Write your answer correct to three decimal places.

9. Write $(1 + \sqrt{5})^3$ in the form $a + b\sqrt{5}$, where a and b are integers.

10. Show that the equation of tangent to the curve $x = \sin 3t$, $y = \cos 2t$, at $t = \frac{\pi}{4}$ is $3y-2\sqrt{2}x+2 =0$